1 * M129 Summer Course 2014-2015

① Sets Numbers

N = Natural = $\{1, 2, 3, 4, \dots\}$

W = Whole = $\{0, 1, 2, 3, \dots\}$

Z = Integers = $\{0, \pm 1, \pm 2, \dots\}$

تكون القسمة منتهية — اعداد دورية = Q = Rational = $\{\frac{a}{b}, a, b \in Z, b \neq 0\}$

$0.32\overline{1}$ — $\frac{2}{5} = 0.4$

IQ = Irrational = قسمة غير منتهية — جذور صماء

$\sqrt{5}, \sqrt{3}, \frac{22}{7}, \dots$

R = All Numbers

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② Domain :-

① Rational function

$f(x) = \frac{h(x)}{g(x)}$

$\Rightarrow$ domain $= R - \{\text{Zero's of } g(x)\}$

Ex: ① $\frac{2x}{3x-6}$

$3x - 6 \neq 0 \Rightarrow$ domain

$x \neq 2$ $\quad R - \{2\}$

Ex ② $\frac{x-3}{x^2+4} = f(x)$

$x^2 + 4 \neq 0 \Rightarrow$ domain

$x^2 \neq -4$ $\quad = R$

② Root function

$f(x) = \sqrt{g(x)}$

domain = intervales of $g(x) \geq 0$

Ex $f(x) = \sqrt{3-x}$

3: $+ \mid -$

$(-\infty, 3]$

Ex $\sqrt{x^2-16}$

$-4 \quad 4$: $+ \mid - \mid +$

$(-\infty, -4] \cup [4, \infty)$

③ Polynomial

$f(x) = x^2 + 5x - 7$

$f(x) = (x-1)^2$

.....

domain $= R$

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② Zero's of any function : The value inwhich = 0

Ex 1 $\quad f(x) = \frac{2x-6}{x^2-16}$

Zero's

$2x - 6 = 0$

$x = 3$

domain

$x^2 - 16 \neq 0$

$x \neq \pm 4$

$R \setminus \{\pm 4\}$

Ex 2

$f(x) = x^2 + 2x - 3$

Zero's $x^2 + 2x - 3 = 0$

$(x-1)(x+3) = 0$

$x = 1$

$x = -3$

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[٢]

نقاط التقاطع

③ The Points of intersection: $\begin{cases} y = \dots \\ y = \dots \end{cases}$ مساواة

Ex $\begin{cases} y = x^2+5x \\ y = 2x^2+6 \end{cases} \Rightarrow x^2+5x = 2x^2+6 \Rightarrow x^2-5x+6=0$

$x = 2$
$x = 3 \Rightarrow$ Points $(2,14)$
نعوض في أي منهما $(3,24)$

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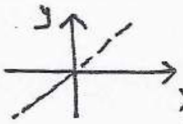
* Some Skills:

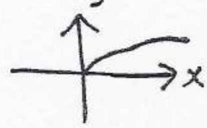
① $(a+b)^2 = a^2+2ab+b^2$

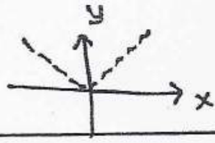
② $a^2-b^2 = (a-b)(a+b)$

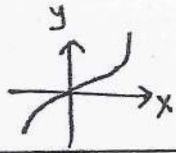
③ Some graphes

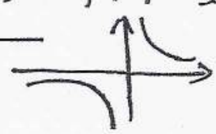
(a) $f(x) = x$ 

(b) $f(x) = x^2$ 

(c) $f(x) = \sqrt{x}$ 

(d) $f(x) = |x|$ 

(e) $f(x) = x^3$ 

(f) $f(x) = \frac{1}{x}$ 

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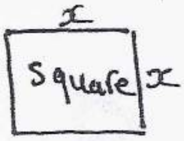
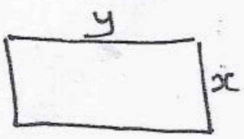
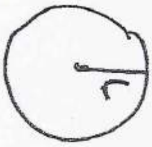
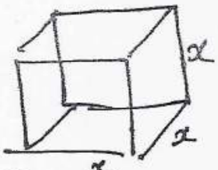
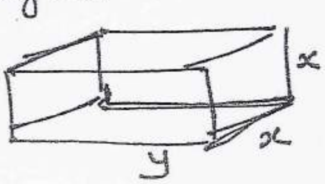
④ shape

		Perimeter	Area
		$4x$	x^2
		$2x+2y$	$x \cdot y$
circle		$2\pi r$	πr^2

		surface	volume
Cube		$6x^2$	x^3
Rectangular		$2x^2+4x \cdot y$	$x^2 \cdot y$

انتبه: (خزان بحجم لو بدون غطاء أو بدون واجهة)

$2x^2+2xy =$ Canvas لو $2x^2+3xy$ بدون غطاء علوي أو واجهة

[٣] operation of function: $+ , - , \times , \div , \circ$

Ex: $f(x) = x^2+6x$

$g(x) = x+3$

* $f \cdot g = x^3+9x^2+18x$

* $f+g = x^2+7x+3$

* $f-g = x^2+5x-3$

* $\frac{f}{g} = \frac{x^2+6x}{x+3} \qquad x \neq -3$

* $f \circ g(x) = (x+3)^2 + 6(x+3)$

$f[g(x)] = x^2+6x+9+6x+18$

$= x^2+12x+27$

$g \circ f(x) =$

$g[f(x)] = (x^2+6x)+3$

$= x^2+6x+3$

Ex: $f(x) = x^2+5$, $g(x) = \sqrt{6-x}$

domain $f = D_f = R$ $\qquad D_g = (-\infty, 6]$

$f \circ g(x) = (\sqrt{6-x})^2 + 5 = 11 - x$

$D_{f \circ g} = (-\infty, 6]$

$g \circ f(x) = \sqrt{6-(x^2+5)}$

$= \sqrt{1-x^2}$

$D_{g \circ f} = [-1, 1]$

Slope : m $\qquad \boxed{y - y_1 = m(x - x_1)}$ equation of Line (tangent)

(a)

$m = \text{slope} = \frac{y_2-y_1}{x_2-x_1}$

When we have two Points $(x_1, y_1), (x_2, y_2)$

(b)

equation $y = mx+b$, Point (x_1, y_1)

$\Rightarrow (y - y_1) = m(x - x_1)$ (Parallel) متوازيان

$\perp \quad (y - y_1) = \frac{-1}{m}(x - x_1)$

(Prependicular) متعامدان

وسوف نتعامل مع المشتقات في الصفحات القادمة

Where Stationary Point $\Leftrightarrow$ Critical Point

$\Leftrightarrow f'(x) = 0$

٤ **Derivatives:**

Rules
1 –
2 –
3 –

① $f(x) = A$ constant $\Rightarrow f'(x) = 0$

② $f(x) = x^n \Rightarrow f'(x) = n\,x^{n-1}$

③ $f(x) = [g(x)]^n \Rightarrow f'(x) = n\,[g(x)]^{n-1} \cdot g'(x)$

Ex:

① $f(x) = x^3 + 2x^2 - 5x + 7 \Rightarrow f'(x) = 3x^2 + 4x - 5$

② $f(x) = (x^2 - 5x)^7 \Rightarrow f'(x) = 7(x^2 - 5x)^6 \cdot (2x - 5)$

③ $f(x) = \dfrac{1}{x^2} + x^2 + 3x \Rightarrow$ نعدل فى صورته

$f(x) = x^{-2} + x^2 + 3x \Rightarrow f'(x) = -2x^{-3} + 2x + 3$

④ $f(x) = \dfrac{1}{\sqrt{x^2 - x}} + 3x^2 - 4$

نعدل فى صورته

$f(x) = (x^2 - x)^{-\frac{1}{2}} + 3x^2 - 4 \Rightarrow f'(x) = -\frac{1}{2}(x^2 - x)^{-\frac{3}{2}} \cdot (2x - 1) + 6x$

*** Derivatives by using The difinition :-** * ٣ أنواع مسائل *

(a)

$f(x) = x^2 - 2x$ نخليها على ٣ خطوات

Solution

$* \; f(x+h) - f(x) =$

$= (x+h)^2 - 2(x+h) - (x^2 - 2x)$

$= x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x$

$= 2xh + h^2 - 2h$

$= h(2x + h - 2)$ عامل مشترك

* نعوض ما وصلنا إليه $\div h$

$\dfrac{\cancel{h}(2x + h - 2)}{\cancel{h}}, \; h = 0$ $\boxed{= 2x - 2}$

(b)

$f(x) = \dfrac{1}{x - 3}$ ٣ خطوات

Solution

$* \; f(x+h) - f(x)$

$= \dfrac{1}{x + h - 3} - \dfrac{1}{x - 3}$ توحيد مقام

$= \dfrac{\cancel{x} - \cancel{3} - \cancel{x} - h + \cancel{3}}{(x + h - 3)(x - 3)}$

$= \dfrac{-h}{(x + h - 3)(x - 3)}$

$* \; \dfrac{-\cancel{h}}{\cancel{h}(x + h - 3)(x - 3)}$ نقسم على h

نعوض $h = 0$

$= \dfrac{-1}{(x - 3)(x - 3)} = \dfrac{-1}{(x - 3)^2}$ *

(c) ٣ خطوات

$f(x) = \sqrt{x + 3}$

Solution

$* \; f(x+h) - f(x) =$

$= \sqrt{x + h + 3} - \sqrt{x + 3}$

بالضرب بالمرافق

$\dfrac{\sqrt{x + h + 3} - \sqrt{x + 3}}{1} \cdot \dfrac{\sqrt{x + h + 3} + \sqrt{x + 3}}{\sqrt{x + h + 3} + \sqrt{x + 3}}$

$= \dfrac{\cancel{x} + h + \cancel{3} - \cancel{x} - \cancel{3}}{\sqrt{x + h + 3} + \sqrt{x + 3}}$

$\div h$ وبعدها نعوض $h = 0$

$\Rightarrow \dfrac{1}{2\sqrt{x + 3}}$ *

Ex: Find The tangent equation of $f(x) = x^2 - 6x + 7$ at $x = 2$

Solution $\acute{f}(x) = 2x - 6$, $\acute{f}(2) = m = 2(2) - 6 = -2$

نعوض في المشتقة

نعوض في أصل الدالة ← $f(2) = y_1 =$

$= (2)^2 - 6(2) + 7$

$= 4 - 12 + 7 = -1$ $\Rightarrow$

$y - y_1 = m(x - x_1)$

$y + 1 = -2(x + 2)$

$y = -2x - 5$

Ex: Find The Second derivatives of المشتقة الثانية

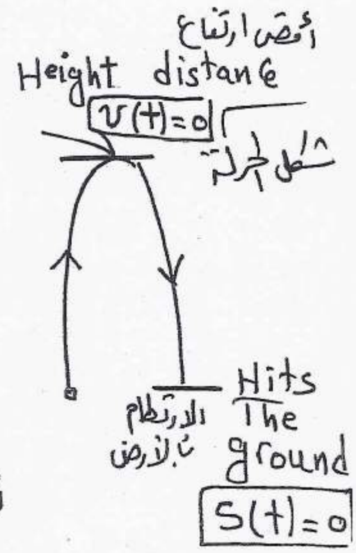
$f(x) = (3x+1)^5$

Solution:

$\acute{f}(x) = 5(3x+1)^4 \cdot 3 = 15(3x+1)^4$

$\ddot{f}(x) = 15 \times 4 (3x+1)^3 \cdot 3$

$= 180(3x+1)^3$



* Position - Velocity - Acceleration

مسافة   السرعة   التسارع

$$s(t) \xrightarrow{\frac{d}{dt}} v(t) \xrightarrow{\frac{d}{dt}} a(t)$$

نشتق المسافة مرة = السرعة ونشتق مرة = التسارع

* **Ex:** $S(t) = 40t - 5t^2 + 100$ meter

$v(t) = 40 - 10t$ m/s

$a(t) = -10$ m/s^2

Find The velocity at $t = 2$

$v(2) = 40 - 10(2)$

$= 40 - 20$

$= 20$ m/s

When Hits The ground

$s(t) = 40t - 5t^2 + 100 = 0$

$t = -2$, $\boxed{t = 10}$

مرفوض

Find The Hieht distance

$v(t) = 0 \Rightarrow 40 - 10t = 0$

$t = 4$

$S(4) = 40(4) - 5(4)^2 + 100$

$= 180$ meter

6

Note

1. x - intersept put $y = 0$
   y - intercept put $x = 0$

Ex: $y = x^2 - 3x - 4$

- x-intercept $y = 0$
  $\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x_1 = 4$, $x_2 = -1$
- y-intercept $x = 0$
  $\Rightarrow y = 0 - 0 - 4 = -4$

2. Number of stationary Points (or critical)

هناك إل عند العدد

$$(f'(x) = 0)$$

Ex: $f(x) = x^5 - 4x^4$

Solution

$$f'(x) = 5x^4 - 16x^3 = x^3(5x - 16) = 0$$

$$x^3 = 0 \quad | \quad 5x - 16 = 0$$

$$\boxed{x_1 = 0} \quad | \quad \boxed{x_2 = 16/5}$$

Number $= 2$: لدينا إثنان من القيم

Application of derivatives:

$f'(x) = 0$

- Interval Increasing _ decreasing تزايد - تناقص
- Local Maximum - Local Minimum Points عظمى صغرى محلية

$f''(x) = 0$

- Interval Concave up _ down تقعر لأسفل - أعلى
- inflection Points إنقلاب

Ex 1

Let $f(x) = x^3 + 3x^2 - 24x + 6$

a) Find $f'(x)$ and $f''(x)$.

$$f(x) = x^3 + 3x^2 - 24x + 6$$
$$f'(x) = 3x^2 + 6x - 24,$$
$$f''(x) = 6x + 6$$

b) Find the intervals on which $f(x)$ is increasing or decreasing.

$$f'(x) = 3x^2 + 6x - 24 = 3\ (x^2 + 2x - 8) =$$
$$(x+4)(x-2), x = -4\ or\ x = 2$$

intervals	(-∞ , -4)	(-4,2)	(2, ∞)
$f'(x)$	+ + + + + +	– – – – – – –	+ + + + + + +
Conclusion	Increasing	decreasing	increasing

c) Find the local maximum and minimum of $f(x)$, if any.
local max (-4, 86)
Local min (2, -22)

d) Find the intervals on which the graph of $f(x)$ is concave up or concave down.

$$f''(x) = 6x + 6 = 6(x+1), x = -1$$

intervals	(-∞ ,-1)	(-1, ∞)
$f''(x)$	-------------------	+ + + + + + +
Conclusion	Concave down	Concave up

Inflection point (-1 , 32)

Ex 2

Let $f(x) = x^4 + 4x^3 + 2$

a) Find $f'(x)$ and $f''(x)$

$$f(x) = x^4 + 4x^3 + 2$$
$$f'(x) = 4x^3 + 12x^2,$$

$$f''(x) = 12x^2 + 24x$$

b) Find the intervals on which $f(x)$ is increasing or decreasing.

$$f'(x) = 4x^3 + 12x^2 = 4x^2\,(x+3) = 0\,, x = 0 \text{ or } x = -3$$

intervals	(-∞ , -3)	(-3,0)	(0, ∞)
$f'(x)$	– – – – – –	+ + + + + +	+ + + + + +
Conclusion	decreasing	increasing	increasing

c) Find the local maximum and minimum of $f(x)$, if any.

No local max

Local min (-3,-25)

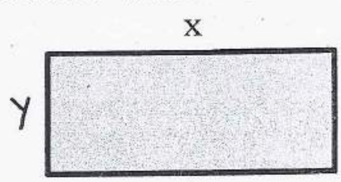
d) Find the intervals on which the graph of $f(x)$ is concave up or concave down.

$$f''(x) = 12x^2 + 24x = 12x(x+2) = 0\,, x = -2 \text{ or } x = 0$$

intervals	(-∞-2)	(-2,0)	(0, ∞)
$f''(x)$	+ + + + + +	– – – – – –	+ + + + + +
Conclusion	Concave up	Concave down	Concave up

Ex:1

A carpenter is building a rectangular room with a fixed perimeter of 360 ft. What are the dimensions of the largest room that can be built? What is its area?



$$2x + 2y = 360\,, y = 180 - x,\ \ A = x.y,$$

$$A = x(180 - x) = 180x - x^2\,,$$

$$A' = 180 - 2x,\ \ 180 - 2x = 0, x = 90\,, y = 90$$

Area = 90 * 90 = 8100 ft^2

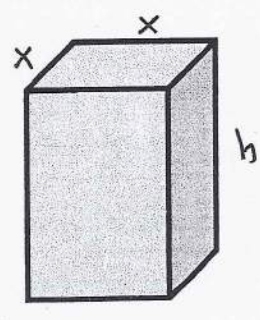
Ex 2

Find two positive numbers such that their product is 75, and so that the first plus three times the second is a minimum.

$$xy = 75, \qquad y = \frac{75}{x}$$
$$s = x + 3y,$$
$$s = x + 3 . \frac{75}{x}$$
$$s = x + \frac{225}{x},$$
$$s' = 1 - \frac{225}{x^2} =$$
$$\frac{x^2 - 225}{x^2} = 0,$$
$$x = \pm 15, so\ x = 15, \qquad y = 5$$

Ex 3

Consider rectangular box with a square base. Find the dimensions of the box for which the volume is 1000 cubic feet and the total surface area for the box is minimal.



$$S = 2x^2 + 4xh, \quad \text{subject to} \quad 1000 = x^2 h$$
$$\Rightarrow S = 2x^2 + 4x \cdot \frac{1000}{x^2} = 2x^2 + \frac{4000}{x}.$$
$$S' = 4x - \frac{4000}{x^2} = 0 \Rightarrow 4x^3 = 4000 \Rightarrow x = 10,\ h = 10,$$

Dimensions : 10feet × 10feet × 10feet.

* chapter(3) (4)(5)

* Rules of derivatives

① Product Rule

: $(f \cdot g)' = f \cdot g' + g \cdot f'$

② Quotient :

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \qquad g \neq 0$$

③ Root : $$\left(\sqrt{f(x)}\right)' = \frac{f'(x)}{2\sqrt{f(x)}}$$

④ Chain Rule :

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

⑤ Implicit Diff.

as We know $\frac{d}{dx} x^2 = 2x$

$$\frac{d}{dx} y^2 = 2y \frac{dy}{dx} = 2yy'$$

(نرمز عن مشتق y بجوارها y')

⑥ $$\frac{d}{dx} e^{f(x)} = f'(x) \cdot e^{f(x)} \qquad e = 2.7182\ldots$$

$$\frac{d}{dx} a^{f(x)} = f'(x) \cdot a^{f(x)} \cdot \ln a$$

Properties of Exponantial Function
$$\begin{cases} a^x \cdot a^y = a^{x+y} \\ a^x \div a^y = a^{x-y} \\ (a^x)^y = a^{x \cdot y} \end{cases} , \quad a^0 = 1 \;\; a \neq 0$$

* Examples

Find $f'(x)$

① $f(x) = (2x+3)(x^2-4x)$

$$f'(x) = (2x+3)(2x-4) + (x^2-4x)(2)$$
$$= 4x^2-2x-12+2x^2-8x = 6x^2-10x-12.$$

② $f(x) = \dfrac{3x-4}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(3) - (3x-4)(2x)}{(x^2+1)^2} = \frac{3x^2+3-6x^2+8x}{(x^2+1)^2}$$

$$= \frac{-3x^2+8x+3}{(x^2+1)^2}.$$

③ $f(x) = \sqrt{x^2+5x} \Rightarrow f'(x) = \dfrac{2x+5}{2\sqrt{x^2+5x}}$

④ $f(x) = \dfrac{(x+1)}{\sqrt{x^2+5}} \overset{\text{حولها}}{=} (x+1)(x^2+5)^{-\frac{1}{2}}$

نتعامل معها حاصل ضرب $f'(x) = (x+1)\cdot \frac{-1}{\cancel{2}}\cdot \cancel{2}x(x^2+5)^{-\frac{3}{2}} + (x^2+5)^{-\frac{1}{2}}\cdot 1$

$$= \frac{-x(x+1)}{(x^2+5)^{\frac{3}{2}}} + \frac{1}{\sqrt{x^2+5}}$$

(5) If $y = \sqrt{u+1}$, $u = 2x^2 \Rightarrow$ Find $\frac{dy}{dx}$

$$\Rightarrow y = \sqrt{u+1} = \sqrt{2x^2+1}$$

$$\Rightarrow y' = \frac{dy}{dx} = \frac{4x}{2\sqrt{2x^2+1}} = \frac{2x}{\sqrt{2x^2+1}}$$

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(6) $4y^3 - x^2 = -5$ use Implicit diff. to determine The slope at The Point $(3,1)$

Solution

(a) Find y'?

$$\Rightarrow 12y^2 y' - 2x = 0$$

$$\Rightarrow y' = \frac{2x}{12y^2} = \frac{x}{6y^2}$$

$$\text{slope} = y'\Big|_{\substack{x=3 \\ y=1}} = \frac{3}{6(1)^2} = \frac{3}{6} = \frac{1}{2}$$

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(7) Find The tangent equation of $\sqrt{x} + \sqrt{y} = 7$ at $(16,9)$

Solution

(a) Find slope $= m$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} y' = 0$$

$$\Rightarrow \frac{1}{\cancel{2}\sqrt{y}} y' = \frac{-1}{\cancel{2}\sqrt{x}}$$

$$\Rightarrow y' = \frac{-\sqrt{y}}{\sqrt{x}}\Big|_{x=16} = \frac{-3}{4}$$

(b) Find equation $(y - y_1) = m(x - x_1)$

$$\Rightarrow y - 9 = \frac{-3}{4}(x - 16)$$

$$y = \frac{-3}{4}x + 13$$

⑧ $f(x) = (x^2-5x)\cdot e^{3x}$

solution

(حاصل ضرب) $f'(x) = (x^2-5x)\cdot 3\cdot e^{3x} + e^{3x}\cdot(2x-5)$

$= e^{3x}\left[3x^2-15x+2x-5\right]$

$= e^{3x}\left[3x^2-13x-5\right]$

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⑨ $y = \dfrac{e^x}{x+e^x}$ Find (a) y' (b) tangent Line at $(0, 1)$

solution

(a) قسمة

$y' = \dfrac{(x+e^x)\cdot e^x - e^x(1+e^x)}{(x+e^x)^2} = \dfrac{xe^x + \cancel{e^{2x}} - e^x - \cancel{e^{2x}}}{(x+e^x)^2}$

$= \dfrac{x\cdot e^x - e^x}{(x+e^x)^2}$ $\Big|_{x=0}$ $= m = \text{slope} = \dfrac{0-1}{(0+1)^2}$

$= -1$

(b) equation

$(y-1) = -1(x-0)$

$\Rightarrow y = -x+1$

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⑩ $x^2\cdot y^2 + 3x^2 = 5y$ Find y' (Implicit)

solution

$(x^2\cdot 2yy' + y^2\cdot 2x) + 6x = 5yy'$

$\Rightarrow 2x^2yy' - 5yy' = -6x - 2xy^2$

$\Rightarrow y'(2x^2y - 5y) = -6x - 2xy^2$

$\Rightarrow y' = \dfrac{-6x-2xy^2}{2x^2y-5y}$

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* Logarithm function :-

Properties : 1- $\ln(xy) = \ln x + \ln y$.

2- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.

3- $\ln x^n = n \ln x$.

4- $\ln e^{f(x)} = e^{\ln f(x)} = f(x)$.

5- $\ln e = 1$

Ex(1) simplify

$\ln 3 - 2\ln x$

$* \; e^{\ln 3 - \ln x^2}$

$= e^{\ln \frac{3}{x^2}}$

$= e^{\ln \frac{3}{x^2}} = \frac{3}{x^2}$ ※

Ex(2) Solve (Find x)

$5\ln(2x) = 8$

$\Rightarrow \ln(2x) = \frac{8}{5} = 1.6$

take e two sides

$e^{\ln(2x)} = 2x = e^{1.6}$

$x = \frac{e^{1.6}}{2} = 2.477$ ※

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* Formula :-

$$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)} .$$

domain $f(x) > 0$

Ex(1) Write as single Logarithm

$$5 \ln x - \frac{1}{2} \ln y + 3 \ln z$$

solution

$$\ln \frac{x^5 \cdot z^3}{\sqrt{y}}$$

Ex(2) (a) $y = \ln(x^2 - 5x + 3) \Rightarrow y' = \dfrac{2x - 5}{x^2 - 5x + 3}$

(b) $y = \dfrac{\ln x}{(x+1)} \Rightarrow y' = \dfrac{(x+1) \cdot \frac{1}{x} - \ln x \cdot (1)}{(x+1)^2}$

$$= \frac{\frac{x+1}{x} - \ln x}{(x+1)^2}.$$

<u>Ex</u>:

$$y = \sqrt[3]{\frac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}} \quad \text{Find } y'$$

<u>Solution</u>:

take ln twosides $\Rightarrow \ln y = \ln \sqrt[3]{\dfrac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}}$

$$\Rightarrow \ln y = \frac{1}{3}\left[2 \ln(x+3) + 4 \ln(2x^2+4) - 6 \ln(5x+1)\right]$$

$$\frac{y'}{y} = \frac{1}{3}\left[\frac{2}{x+3} + 4 \cdot \frac{4x}{2x^2+4} - 6 \cdot \frac{5}{5x+1}\right]$$

$$\Rightarrow y' = \frac{1}{3}\left(\sqrt[3]{\frac{(x+3)^2 \cdot (2x^2+4)^4}{(5x+1)^6}}\right)\left[\frac{2}{x+3} + \frac{16x}{2x^2+4} - \frac{30}{5x+1}\right]$$

chapter(5)

* Exponential growth and decay model

$$e^{kt} \begin{cases} \text{growth if } k>0 \\ \text{decay if } k<0 \end{cases}$$

* The Function $P(t) = P(t_0) \cdot e^{kt}$

satisfies The differential equation

(Fast) $P'(t) = kP(t)$

where
$P(t_0)$ = Initial Population
k = Constant

Ex(1)
In April 1994 The Population of a small town was estimated 2500 People. and $A(t) = Ce^{kt}$, k is increase rate
IF There was 3900 in April 1999

Find (a) rate inkease(k)?

$P(t_0) = 2500$, $P(t) = P(5) = 3900$, $t = 5$ (94 → 99

$$\Rightarrow 3900 = 2500 \cdot e^{k(5)} \Rightarrow e^{5k} = \frac{3900}{2500}$$

$$\Rightarrow e^{5k} = 1.56 \Rightarrow 5k = \ln(1.56) \Rightarrow$$

$$k = \frac{\ln(1.56)}{5} = 0.0889$$

(b) Writ A(t)? $\Rightarrow A(t) = 2500 \cdot e^{0.0889t}$.

(c) $A(10) \Rightarrow A(10) = 2500 \cdot e^{0.889 \times (10)} = 6087.8$

Ex(2) (Radioactive decay)

Radium is used in cancer radiotherapy. Let $P(t)$ be the number of grams of radium after t year.

$P(t)$ satisfy
$$P'(t) = -0.00043\,P(t) \rightarrow (\star)$$
$$P(0) = 12$$

(a) Find the formula of $P(t)$

$$P(t) = 12 \cdot e^{-0.00043t}$$

(b) initial condition $= 12$, decay constant $= -0.00043$

(c) Find the radium after 943 years?

$$t = 943 \Rightarrow P(t) = 12 \cdot e^{-0.00043(943)} \simeq 8 \text{ grams}$$

(d) how fast when one gram remains?

نعوض في $(\star)$

$$\Rightarrow P(t) = 1 \Rightarrow P'(t) = -0.00043(1) = -0.00043$$

i.e. rate $= 0.00043$